

Digital simulation of phonograph tracking distortion*

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ABSTRACT

Phonograph tracking distortion results from the misalignment of a playback cartridge with respect to the cutting head. While it has been researched for decades, it remains a source of mystery: it has never been accurately isolated or simulated. A simulation of horizontal and vertical tracking distortion of extremely high quality is presented, operating on the principle of phase modulation of PCM digital audio, allowing tracking distortion to be evaluated, in isolation, with real musical content. In this context, tracking distortion is equivalent to digital audio sampling jitter, with the jitter spectrum equal to the signal spectrum. Implications of this connection, as well as simulation accuracy, preliminary listening test results, and potential applications are discussed.

1. A REVIEW OF TRACKING DISTORTION

Phonograph recording operates by cutting a groove onto a moving surface. The cutter head moves laterally and vertically in proportion to the input signal, thus modulating the position of the groove on two axes orthogonal to the axis of movement. Phonograph playback typically uses a stylus-tipped cantilever which tracks the horizontal and vertical modulations of the groove. The cantilever's motion is converted to an electrical signal, ideally matching that which was originally recorded.

The position of the cutting head relative to the motion of the recording surface defines two angles, the vertical modulation angle (VMA) and horizontal modulation angle (HMA).¹ Similarly, during playback, the angle of the cantilever with respect to the axis of motion of the record defines a horizontal tracking angle (HTA) and vertical tracking angle (VTA). The modulation and tracking angles define a change in coordinate axes: a signal modulated along that axis, rather than (for instance)

being applied solely to the horizontal and vertical axes relative to the record surface, is also modulated onto the axis of groove motion — a form of crosstalk between the modulation axis and the time axis. When the tracking angle matches the modulation angle, this modulation cancels out. When the angles differ, the difference is referred to as tracking (angle) error, and the distortion produced is tracking (angle) distortion.

The works of Löfgren and Baerwald remain primary analyses for the geometric description and analysis of the distortion for the case of simple signals [1] [2]. A modulated signal $y(s) = y_0 \sin \phi(s) = y_0 \sin \omega t$, originally the velocity-sensitive electrical signal $v(s) = dy/ds = v_0 \cos \omega t = y_0 \omega \cos \omega t$, when tracked, is transformed into²:

$$\begin{aligned} Y(s) &= y_0 \sec \eta \sin \psi(s) \\ \phi &= \psi - \epsilon \sin \psi \\ \epsilon &= \frac{y_0 \omega}{r \Omega} \tan \eta = \frac{v_0 \tan \eta}{r \Omega} \end{aligned} \quad (1)$$

*This is a revised version of the preprint presented at the 127th AES convention in October 2009, and available from the AES. That preprint remains the authoritative version of this text. Here, the copyrighted AES logo and other convention livery are removed, but the contents of the paper are unmodified.

¹That stereo records are cut 45 degrees rotated from either axis is not of concern; 45-45 and lateral/vertical representations are interchangeable.

²This paper (and the simulation it describes) ignores the scale factor $\sec \eta$, following Cooper's example of the use of the normalized skew transform [3]. That is, the optimum tracking angle for given modulation angle η is not η , but instead $\tan^{-1} \sin \eta$. This detail is generally not a salient matter for the purposes of this paper — the resulting error for η only reaching 5% for $\eta = 19^\circ$ and having no impact on the nature of the distortion — but may be considered when regarding physical manifestations of tracking distortion.

Löfgren and Baerwald (the latter after making an entertaining connection with celestial mechanics) obtain the solution

$$Y = \frac{v_0}{\omega} \sec \eta \sum_{n=1}^{\infty} \frac{J_n(n\epsilon)}{1/2n\epsilon} \sin n\omega t$$

$$V = v_0 \sec \eta \sum_{n=1}^{\infty} \frac{J_n(n\epsilon)}{1/2\epsilon} \cos n\omega t \quad (2)$$

Because all cutting heads are mounted on a linearly articulated carriage, the HMA is effectively zero, and minimizing HTA also minimizes horizontal tracking distortion. In contrast, the VMA (and thus the optimal VTA) is much greater than zero³. Fig. 1 describes the relevant geometry for horizontal tracking, as Fig. 2 does for vertical tracking.

Two substantial gaps remain in the theory of tracking distortion. First, its evaluation has only been made with any kind of accuracy for sums of sinusoids; a high-quality approximation for arbitrary musical signals is not attested in the literature. Second, the interaction of several sources of distortion in a physically realizable phonograph, including tracking distortion, makes the study of tracking distortion’s audibility threshold intractable. Thus, what one would expect to be a rather simple question to answer — “*how small a tracking error is necessary for highest performance?*” — has never been conclusively answered.

Such a question remains, to some degree, pertinent today. High-end audio firms continue to supply devices to minimize tracking distortion, from linear tracking tonearms to alignment protractors. This is often accompanied with claims of astoundingly low audibility thresholds — one high-end authority has asserted that a VTA error of 12 *arcseconds* is audible. In the larger audio market, the continued and improbable popularity of vinyl as a new music format, with unit sales growth of 200% over the last two years [5], keeps relevant the discussion of best practices of turntable configuration and maintenance for mainstream consumers. In both cases, the existence of an authoritative audibility threshold for tracking distortion may help quality-conscious consumers make more informed decisions. While the

³with some allowance for nonlinear effects such as lacquer springback [4].

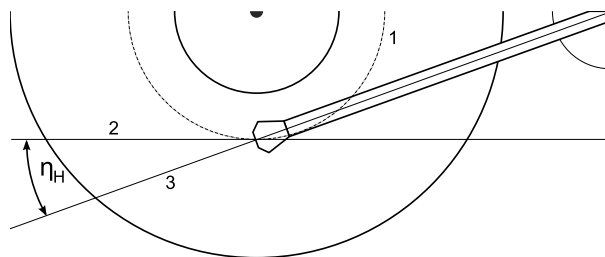


Fig. 1: Diagram illustrating geometry of horizontal tracking error η_H ; this is also the HMA. This angle is deliberately exaggerated for the purpose of illustration. 1: Path of groove; 2: Tangent of groove at point of stylus contact; 3: Axis of cantilever. η_H is the angle between lines 2 and 3.

interaction of tracking distortion with other distortions make such thresholds harder to apply directly to real world recommendations, by no means do they make them inapplicable.

2. A DIGITAL SIMULATION

2.1. Principles of operation

Cooper insightfully commented that tracking distortion is a form of phase modulation [7]. Indeed, much literature on tracking and tracing distortion in the 1960s contains several operational models involving variable-tap analog delay lines. Such contraptions seem comical today, given that such delays are trivial to implement with PCM digital audio. The interpretation of tracking error in this manner — as *strictly* a form of modulation along the time axis — allows its numeric approximation using the same techniques used to simulate sampling jitter, a field with considerable modern interest [9]. Indeed, as Dunn pointed out, “the effect of sampling jitter is to phase modulate the signal” [8]. This meaning of phase modulation in the context of sampling jitter is precisely the same meaning that Cooper uses to describe tracking distortion.

2.2. Implementation

All of this immediately suggests the implementation of such a distortion simulator with arbitrary PCM digital audio signals. The magnitude of the phase modulation, represented by Baerwald as the distortion parameter ϵ [1], is represented digitally by the distortion factor E . It is rather straightforward to

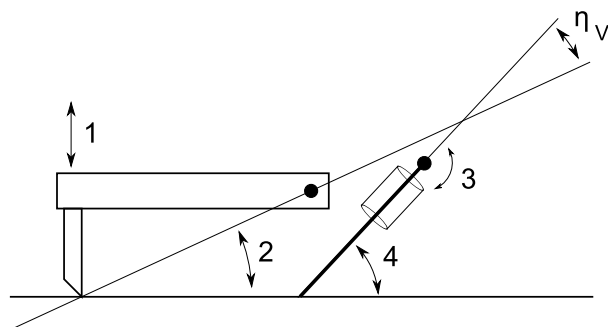


Fig. 2: Diagram illustrating geometry of vertical tracking error η_V (deliberately exaggerated for the purpose of illustration). 1: Cutting head with indicated axis of motion; 2: Vertical modulation angle (VMA) defined by line from cutting stylus contact point to axis of rotation; 3: Playback cantilever with indicated axis of motion; 4: Vertical tracking angle (VTA) defined by line from stylus contact point to axis of rotation. η_V is the difference between angles 2 and 4.

describe the simulation algorithm:

Algorithm: Simulate horizontal and vertical tracking distortion on PCM audio signals.

Inputs: Original stereo signal $\langle L(t), R(t) \rangle$; horizontal tracking distortion factor E_H ; vertical tracking distortion factor E_V .

Output: Distorted stereo signal $\langle L_S(t), R_S(t) \rangle$.

1. Upsample the signal so that the harmonics caused by the tracking distortion do not alias back into the audible band. In practice an 8x oversampling appears acceptable.
2. Convert to unequalized amplitude domain: apply reverse RIAA equalization and numeric integration. At an acceptably high oversampling ratio, the naive implementations of RIAA equalization (via prewarped bilinear transform) and of integration (via first-order difference equation) are acceptable.
3. Apply Cooper's skew operator [6]: Perform a time-varying resample operation, where each channel sample $L(t), R(t)$ is moved to a new location $L_S(t + \Delta t) = L(t), R_S(t + \Delta t) = R(t)$. The magnitude of the skew Δt is equal to the

sum and difference signals scaled by the horizontal and vertical distortion factors, respectively:

$$\Delta t = E_H \left(\frac{1}{\sqrt{2}} \right) (L(t_0) + R(t_0)) \\ + E_V \left(\frac{1}{\sqrt{2}} \right) (L(t_0) - R(t_0))$$

The distortion factors E_H, E_V are the PCM analogues of the distortion factor $\tan \eta$, adapted to handle both horizontal and vertical tracking distortion.

4. Convert to RIAA-equalized velocity domain: apply forward RIAA equalization and numeric differentiation.
5. Downsample to yield $\langle L_S(t), R_S(t) \rangle$.

This algorithm was implemented in LabVIEW 8.2.1 with the Digital Filter Design Toolkit. Note that unlike Hawksford's work with digital jitter simulation, fewer computational shortcuts can be made when applying the skew, because the jitter amplitude (in seconds) is far larger than the sampling period. However, the fundamental mathematical operation is identical.

Note that the integration adds an extreme sensitivity to DC and very low frequency levels, in effect converting them to frequency drift. While any level of DC removal for input signals subject to listening tests should make this effect inaudible, it must be taken into account when performing certain numeric tests on periodic signals, such as null tests against theoretically derived waveforms.

Mistracking (where the stylus physically loses contact with the groove walls) is not considered in this simulation. If mistracking is simulated — that is, if the skewed samples are not monotonic — the simulator forces monotonicity by deleting samples with negative time offset. This tends to overestimate velocities and underestimate the duration of the mistracking, and therefore likely underestimates the magnitude of the distortion. Of course, this situation should never occur with high-fidelity playback in the first place.

Figures 3(a)–3(c) describe the simulation process graphically. After inverse RIAA equalization and integration, the waveform is literally “skewed” in one

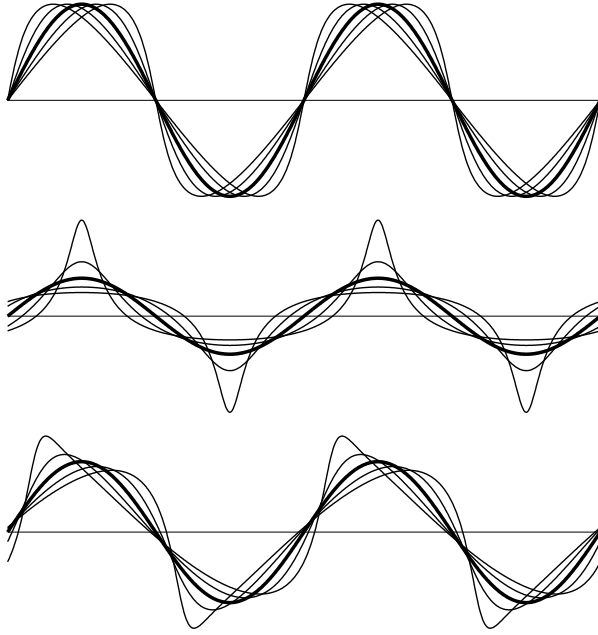


Fig. 3: Sinusoids with varying levels of tracking distortion: (a) path traced by stylus; (b) velocity-sensitive magnetic cartridge output; (c) with RIAA equalization. (Distortion levels shown are exaggerated beyond realistic levels.)

direction or another, in proportion to the amount of tracking error (Fig. 3(a)). Then the waveform is differentiated, just like the velocity-sensitive output of the phono cartridge (Fig. 3(b)). Finally, RIAA equalization is applied to the waveform as in the phono preamp (Fig. 3(c)).

2.3. Relationship to record geometry

The simulation algorithm does not reference real-world parameters that influence distortion, such as tracking angle, rotational velocity, etc. For the simulation to relate to past work and real-world results, the 0dBFS level of a PCM signal must in some way relate to the 0dB level of a vinyl record. While $\tan \eta$ and E superficially represent the same quantity, they are not the same thing: E represents the number of seconds a waveform is shifted if the waveform value is equal to 1, while $\tan \eta$ represents the distance a modulated groove is shifted as a ratio of the modulation width. A more accurate analogy would be between E and ϵ (and as will be soon seen, the choice

η_H	H.T. error ($^\circ$)
η_V	V.T. error ($^\circ$)
E_H	Digital H.T. distortion factor (s)
E_V	Digital V.T. distortion factor (s)
r	Groove radius (cm)
$L(t)$	Left channel signal
$R(t)$	Right channel signal
$A_d(t)$	PCM modulated signal (s)
$V_d(t)$	PCM velocity signal (unitless)
$A_a(t)$	Groove modulation signal (cm)
$V_a(t)$	Groove velocity signal (cm/s)
P	PCM waveform peak
V	Analog velocity peak (cm/s)

Table 1: Some definitions for horizontal tracking (H.T.) and vertical tracking (V.T.) distortion.

of symbol for E is apt) — but it is still not entirely clear what the specific relationship is between the two quantities.

To resolve this ambiguity, the same signal is synthesized in both the analog and digital domains, and the magnitude of the skew in both domains is set to be equal, allowing the solution of E . The signal in question is a sine wave of angular frequency ω . For the purposes of this analysis, quantization error and sampling rates are neglected, and the simulated signal is treated as a real function.

In the simulated (digital) domain, it has velocity peak P , which is the simple peak of the waveform: $V_d(t) = P \sin \omega t$. This is integrated, and then skewed with a given skew factor E . The peak of the signal shifts by $S_d = -PE/\omega$.

In the analog domain, this wave has velocity peak V . This is the peak modulation level of the recording and is typically measured in cm/s: $V_a(t) = V \sin \omega t$. This is integrated into $A_a(t) = -\frac{V}{\omega} \cos \omega t$. This signal is to be distorted according to a given distortion factor $\tan \eta$ — or more specifically, ϵ , which is in the proper units of time. For a groove rotating at a particular angular velocity Ω at radius r , the groove will move a distance of Ωr in 1 second. Therefore, the magnitude of the skew is $S_a = -V \tan \eta / \omega \Omega r$. The magnitude of the skew is asserted to be equal in both domains, yielding $E = V \tan \eta / P \Omega r$. A comparison with (1) yields the very intuitive

$$\epsilon = EP$$

P	33rpm				45rpm			
	1	0.5	1	0.5	1	0.5	1	0.5
r	6	6	15	15	6	6	15	15
E	η ($^\circ$)							
0.001	0.12	0.06	0.30	0.15	0.16	0.08	0.40	0.20
0.00178	0.21	0.11	0.53	0.26	0.29	0.14	0.72	0.36
0.00316	0.38	0.19	0.94	0.47	0.51	0.26	1.28	0.64
0.00562	0.67	0.33	1.67	0.83	0.91	0.46	2.27	1.14
0.010	1.19	0.59	2.97	1.48	1.62	0.81	4.04	2.02
0.0178	2.11	1.06	5.27	2.64	2.88	1.44	7.17	3.60
0.0316	3.75	1.88	9.30	4.68	5.11	2.56	12.59	6.37
0.0562	6.65	3.33	16.24	8.29	9.03	4.54	21.67	11.23
0.100	11.71	5.92	27.40	14.53	15.79	8.05	35.25	19.46
0.178	20.26	10.46	42.70	24.77	26.72	14.12	51.52	32.17
0.316	33.23	18.14	58.60	39.32	41.78	24.07	65.88	48.16
0.562	49.37	30.23	71.05	55.53	57.82	38.47	75.87	63.28
1.000	64.25	46.03	79.08	68.90	70.52	54.73	81.95	74.20

Table 2: Relationship between digital tracking distortion factor E and tracking angle error η for different playback configurations. $V=10\text{cm/s}$. Values in bold are less than what is obtainable with the optimal alignment of a 9 inch tonearm ($r = 6\text{cm}$: 0.84° , $r = 15\text{cm}$: 2.26°).

Note that this derivation applies to either the lateral or the vertical signal domain, and not to the left or the right channel alone. A signal that is constructed from the left and right channels yields a signal amplitude in the lateral or vertical domain according to the groove geometry: the mono amplitude is $(L + R)\sqrt{2}$ and the vertical amplitude is $(L - R)\sqrt{2}$. That is, a stereo WAV file containing equal sine waves in each channel with peak 1 represents a mono signal with $P = \sqrt{2}$, not $P = 1$. Also note that existing literature specifying theoretical distortion levels typically does not take the RIAA equalization curve into account.

2.4. Validation

First, consider a real-world tracking scenario in the optimal alignment⁴ for a 33rpm record with $r=5.7\text{cm}$, $V=10\text{cm/s}$, and $\eta = 1.393^\circ$. The simulator was configured with $E = 0.008642$ with RIAA equalization disabled. The ratio of the second harmonic to the fundamental in the simulation output was found to be 1.219%. The exact value, using

⁴Also known as the “Baerwald, “Löfgren A”, “Stevenson B”, and possibly the “Kessler” and “Pisha” alignments, all of which describe the same alignment originally discovered by Löfgren, and result in the lowest weighted tracking error [10] [11].

(2), is 1.2221%, a difference of -51.8dB . Additionally, a typical treatment of tracking distortion for the purposes of tonearm alignment and design utilizes the successive approximations $\tan\eta \approx \eta$ and $\text{THD} \approx V_2 = \epsilon$, yielding a second harmonic amplitude of 1.2216% (note that this is an amplitude and not relative to the fundamental).

Second, a null test was carried out with simulated output against a theoretically predicted waveform. Distortion was applied (skew, integration and differentiation only) to a 100hz sine wave at $E = 0.5$, $P = 0.5\sqrt{2}$ and $F_s = 88200\text{ Hz}$. Then the theoretical waveform was computed from (2), with $\epsilon = 1/(2\sqrt{2})$. The two waveforms were subsample-aligned by hand at $t = 0$, requiring a time shift of $0.129727/F_s$ in the construction of the theoretical waveform. When the two signals were nulled, the *peak* level of the difference from $t = 0\text{ s}$ to $t = 2\text{ s}$ was found to be -117dB , with no frequency component above -120dB , and perhaps only six components above -144dB .

3. LISTENING TEST

3.1. Configuration

This manipulation and interpretation of the distort-

tion parameter E allows audibility testing in the digital domain to directly relate to the analog domain. To that effect, some listening tests were conducted to establish rough bounds of audibility of tracking distortion when simulated in isolation. Obviously, pathological and unrealistic test cases exist for which any level of distortion is audible, no matter how small. For instance, two ultrasonic tones spaced 3kHz apart and with amplitudes near 0dB generate audible distortion at any nonzero distortion factor; but such a groove would likely prove uncuttable, untrackable, and unrepresentative. The test should use samples that are both adequately sensitive and reasonably representative of the music available on the format. Three samples were selected for this test:

- (piano) Track No. 39sm from the EBU SQAM sample set [12]
- (quartet) Track No. 48 from the EBU SQAM sample set [12]
- (tone) 220hz sinusoids either in phase (mono test) or out of phase (stereo test) across two channels

These samples were normalized to a peak level of 0dBFS and run through the simulator at varying values of either horizontal or vertical distortion, from 0.0001 to 1 in both signs in quarter-decade increments, plus 0: 0, ± 0.0001 , ± 0.000178 , ± 0.000316 , ..., ± 1 . After simulation, each set of either horizontal or vertical simulation results for a particular sample were normalized. All pre- and post-processing was performed with Audacity exporting 16-bit WAVs with triangular shaped dither.

Listening tests were performed by the author and a friend to establish some upper bounds on the limits of inaudibility. Each tester listened in a home listening environment to the best of his abilities: one listener used ABX software on a PC and listened through headphones, while the other listener performed sighted listening tests on his personal speaker system. Insofar as this is not meant to be a conclusive test of audibility — merely a first cut to establish upper bounds of what the audibility limit might be — these protocols are acceptable.⁵

⁵The author believes that the novelty or lack of public

3.2. Results

These results — with a positive audibility result at $|E| = 0.0178$ for a synthetic signal — suggest upper bounds for the limits of audibility of horizontal and vertical tracking distortion as follows⁶:

- horizontal and vertical tracking angle error must be no higher than 2.11° at the inner groove;
- at 45rpm, horizontal and vertical tracking angle error must be no higher than 2.88° at the inner groove;
- modulation velocity must be no higher than roughly 25 cm/s;
- at 45rpm, the modulation velocity must be no higher than 34 cm/s.

If the audibility limit is restricted to the tests involving samples of live music as opposed to artificial signals, $|E| = 0.100$ and the implied limits are modified as follows:

- horizontal and vertical tracking angle error must be no higher than 11.71° at the inner groove;
- at 45rpm, horizontal and vertical tracking angle error must be no higher than 15.79° at the inner groove;
- modulation velocity must be no higher than roughly 142 cm/s;
- at 45rpm, the modulation velocity must be no higher than 193 cm/s.

3.3. Discussion

The maximum horizontal tracking error of a 9" tonearm in the optimal alignment is 0.8° ; the maximum vertical tracking error was estimated in 1984

knowledge about a distortion modeling technique generally precludes its use in a listening test that will be accepted as authoritative. A rigorous double-blind study of tracking distortion will be conducted in the future.

⁶Assuming an inner groove radius of 60mm, an outer groove radius of 146mm, $P=0$ dBFS, a tonearm with a 228.6mm effective length in the optimal alignment, a peak velocity of 10cm/s, and playback of a 300mm, 33rpm record unless otherwise specified.

Sample	A +	A -	B +	B -
Piano Horizontal	0.178	-0.100	0.100	-0.100
Piano Vertical	0.316	-0.316	0.316	-0.316
Quartet Horizontal	0.316	-0.178	0.178	-0.178
Quartet Vertical	0.100	-0.178	0.100	-0.100
Tone Mono	0.0316	-0.0523	0.316	-0.316
Tone Stereo	0.0178	-0.0178	0.178	-0.178

Table 3: Observed audibility limits (minimum E) for listeners A and B, with distortion in both positive and negative polarities.

with period pickups and cutting heads to be 15° [13]. Thus, the audibility of gross amounts of vertical tracking distortion (roughly 15°) is affirmed. However, it is unclear how often this case occurs in practice. Regarding horizontal tracking distortion, these results cannot discount the possibility (but certainly do not state with authority) that a properly aligned 9" tonearm might already have inaudible levels of tracking distortion. The measured audibility limit of horizontal tracking distortion, excluding results with artificial signals, is equivalent to 11.71° .

A subjective listening survey conducted by Cooper seems to corroborate these results [14]. Given vertical tracking errors of $20\text{-}30^\circ$, records cut with a vertical level peak of -12dB ref. 10cm/s were subjectively perceived as having only slight distortion. Being generous by assuming a signal at the inner groove of a 33rpm disc and a 30° error, this distortion can be estimated as $E = 0.098$. This is astonishingly close to the $E = 0.1$ audibility threshold obtained here — all the more so that Cooper’s test likely involved significant amounts of tracing distortion.

In the context of vertical tracking distortion, these results suggest that, for some listeners, VTA adjustment *may* be necessary to ensure the complete inaudibility of vertical tracking distortion. For some configurations of turntable and music, it may not be necessary. If a “sweet spot” exists for VTA alignment for non-synthetic music, it might not be smaller than perhaps 10° , which may be wide enough for many (but not all) playback situations. However, as was mentioned earlier, VTA errors up to 15° may

⁷Maximum vertical tracking error may be as high as 40° for early stereo records which were not cut with lacquer playback in mind.

exist with modern records, and as vertical modulation angles (VMAs) have varied by far higher than 10° , some records will inevitably fall outside such a margin.

That said, the commonly accepted wisdom regarding tracking distortion seems at distinct odds with these results, and would strongly assert that small differences in alignment make a large impact on inner groove distortion. In response, some points must be emphasized.

- The present listening test results are not authoritative, and future testing may uncover far lower audibility thresholds of real-world significance.
- Tracking error also has an important role in tracing distortion, insofar as the misalignment of a stylus causes the two groove contact surfaces to be out of alignment. Such tracing distortion, not in the scope of this paper, may cause a distortion significant enough to justify commonly accepted wisdom. That is, while tracking *distortion* may play a relatively minor role, tracking *error* might not.
- Common wisdom regarding vinyl has been largely unchallenged by blind listening tests, in comparison to many other fields; it may change with improved testing.⁸

4. ADDITIONAL COMMENTS

⁸While it might seem outlandish to suggest that a placebo effect might play a role in so distortion-laden of a medium as vinyl, that possibility remains significant and cannot be ruled out at this time. Ironically, such a result would support the claim that vinyl is capable of the highest music fidelity.

4.1. Establishment of more rigorous audibility thresholds

With additional testing, including more sensitive problem samples and a greater number of testers and listening environments, authoritative audibility limits for E_H and E_V may be obtained. These limits may have substantial implications for phonograph design, purchase, and maintenance, and for the recording of records.

4.2. A possible distortion asymmetry due to polarity

Tracking distortion can skew the waveform in one of two different directions depending on the polarity of E . Subjectively, it was observed during testing that the sound of the distortion was substantially different depending on its polarity (although no blind test was conducted on the matter). As shown in Fig. 3.2, detected thresholds in the negative polarity were consistently equal or lower than those in the positive polarity. It is speculated that, if additional testing confirms a consistently lower threshold for one polarity over the other, cartridge alignment techniques may take advantage of this asymmetry by shifting tracking error towards a greater magnitude in the less sensitive polarity.

4.3. Distortion cancellation

Tracking distortion is, with some restrictions, linear and completely reversible [3]. If E can be estimated in a real record, the tracking distortion present in the recording may be eliminated. E may be estimated across the whole disc with knowledge of the cartridge alignment, and at small timescales, might also be derived from the low frequency content of a record (insofar as it reflects low-frequency cantilever deflections via the cartridge compliance).

However, the high audibility thresholds of tracking distortion so far obtained ought to make this correction unnecessary. If the audibility thresholds for E are reduced by perhaps 1–2 orders of magnitude — so that tracking angle differences within a fraction of a degree are shown to be audible in blind testing — tracking distortion cancellation may become a more viable technique for improving sound quality.

4.4. Simulation of tracing distortion

Cooper notes that tracing and tracking error can be simulated through similar means [6]. In fact, for a spherical stylus profile, tracing distortion is merely

a skew transform with the skew magnitude proportional to the sine of the arctangent of the groove slope. Adapting the existing distortion simulation to handle basic forms of tracing distortion is thus a trivial exercise, and might in theory be extended for more esoteric concerns such as stylus rake angle (SRA) and plastic/elastic deformation. Thus, all other potential uses of this simulation, as they apply to tracking distortion, also apply to tracing distortion as well. Specifically, audibility bounds may be established for various stylus parameters, including contact radius, stylus rake angle, etc. Furthermore, a tracing distortion simulator may be constructed of potentially higher quality than the power-series-based simulations described in the literature [16]. In principle, such a skew-based distortion should deliver more consistent results.

4.5. Other uses

Tracking distortion simulation may be useful for artistic or antiquing purposes, as is occasionally used in modern music production. However, the range between audibility and outright mistracking is surprisingly small, which may limit its usefulness.

4.6. Tracking distortion and sampling jitter

The connection between tracking error and sampling jitter was alluded to earlier. This will now be made explicit. Suppose that a tracking distortion is applied to a signal $f(t)$ yielding $g(t)$. We have [6], first in the general case and then for the sinusoidal input,

$$\begin{aligned} g(t) &= f(t + \epsilon g(t)) \\ &= f(t + \epsilon f(t + \epsilon f(t + \dots))) \\ f(t) &= \sin \omega t \\ X(t) &= \epsilon \sin \omega (t + \epsilon \sin \omega (\dots)) \\ g(t) &= \sin \omega (t + X(t)) \end{aligned} \quad (3)$$

Compare to the equation for a single tone with applied sampling jitter at a single frequency, from [8] with a substitution of variables,

$$\begin{aligned} Y(t) &= \epsilon \sin \omega_j t \\ g(t) &= \sin \omega_i (t + Y(t)) \end{aligned} \quad (4)$$

The connection should be apparent: while the continued expansion present in (3) does not exist in (4), both are clearly pure perturbations of the time axis.

The difference between the two stems from the magnitude of the tracking error perturbation being proportional to the magnitude of the signal itself. Such a relationship prevents an exact analysis, but it is comparatively easy to approximate in the PCM domain.

In principle, the operation of a tracking distortion simulator differs from a sampling jitter simulator in four respects. First, obviously, the modulation waveform is to be generated from the input signal, as opposed to an independent data source. Second, the modulation magnitudes may be far larger, necessitating an abandonment of certain optimizations assuming jitter amplitudes close to the original sample times [9]. Third, RIAA equalization and numeric differentiation must be applied (and reversed) to the input signal to match the electronic circuits and velocity-sensitive transducers present in standard recording and playback environments.

Thus, it is quite inaccurate to suggest that digital audio jitter is a form of distortion *absent* in analog systems, particularly in vinyl. Rather, the most important distinctions to be made between the two distortions relate to their respective spectra: the phase modulation spectrum of tracking distortion is entirely coincident with the original signal, generating intermodulation products coincident with other sources of harmonic distortion, and of an unusually low order at that: the distortion level decreases geometrically: -40dB , -80dB , -120dB , etc. The distortion levels at the 5th and higher orders will not break the noise floor except under extreme circumstances.

In comparison, sampling jitter spectra can be (and often are) unrelated to the original signal spectrum, leading to a greater risk of generating aharmonic distortion. The existence of RIAA equalization and velocity transduction with tracking distortion is also an important difference. At the same time, while an equivalence exists between tracking distortion and sampling jitter, such is *not* the case for other forms of jitter distortion, such as line-induced jitter, or frequency-induced amplitude modulation present in some converter designs [8].

In short, the discussion of digital audio jitter alongside analog audio requires nuance with respect to the spectrum of the phase modulation and the order of the distortion. Regardless, it must be reemphasized

that *the principle of operation of both tracking error and sampling jitter is the same.*

5. CONCLUSION

The theory of tracking distortion is revisited and reformulated in the context of digital signal processing. Upper bounds on audibility limits are suggested based on preliminary testing. At present, given a common industry practice of ca. 10cm/s reference recording levels and 9" playback tonearms in the optimal alignment, the presently obtained limits can only affirm the audibility of vertical tracking error at its most extreme ranges; they otherwise cannot exclude these practices from the region of inaudibility. More work is necessary to establish a more rigorous lower bound on tracking distortion audibility, and to estimate the audibility of tracing distortion, before firmer conclusions may be drawn regarding the importance of tracking error.

Ultimately, this analysis of tracking distortion suggests both good and bad news for vinyl lovers. On one hand, it perhaps may not be generally known that vinyl records are, in a very literal sense, plagued by a time distortion largely equivalent to sampling jitter. Under no circumstances should the myth be perpetuated that digital music is intrinsically inferior to vinyl merely because of the existence of digital audio jitter. On the other hand, the nature of this distortion remains intrinsically different (and likely less audible) from jitter, due to its more harmonic nature. And the difficulty observed in distinguishing sub-degree tracking angle differences suggests the possibility that tracking distortion — long considered a fundamental limitation of the vinyl format — may very well not be as significant a problem as commonly believed.

The harmonic nature of tracking distortion complicates its comparison to digital audio jitter. Nevertheless, this provides an additional data point for the ongoing discussion of the audibility of jitter.

Regardless of the results of this paper, improving phonograph playback should always take into account all known forms of distortion. However, it is the author's hope that this paper will catalyze further interest in the field of phonograph digital analysis and simulation, and that such work will eventually result in higher sound fidelity at lower cost to the discerning listener.

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